# Mathematics Boot Camp for Incoming Ph.D. Students

Summer 2025 · July 28 – August 15

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| **Instructor** | Ayixunbati (Eisen) Nuerlantai |
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| **Dates** | July 28 (1–4 PM); July 29 – August 14 (9 AM – 1 PM, weekdays); August 15 (final exam) |
| **Location** | GC 5490 |
| **Format** | Lecture and problem-solving practice, with a 10–15 minute break halfway through each session |
| **Assessment** | Diagnostic quiz (July 28), problem sets, and final exam (August 15) |

## Course Overview

This boot camp prepares incoming Ph.D. students in economics with the essential mathematical tools and intuition required for first-year coursework in microeconomics, macroeconomics, and econometrics. It refreshes and strengthens foundational concepts in algebra, calculus, linear algebra, optimization, probability, and dynamic modeling, with an emphasis on applications relevant to economics.

## Learning Objectives

By the end of the boot camp, students will be able to:

- Solve optimization problems using calculus and linear algebra techniques.
- Understand and manipulate functions in single- and multivariable settings.
- Work confidently with matrices, vector spaces, and eigenvalue problems.
- Apply basic dynamic systems concepts (difference equations, phase diagrams).
- Understand probability distributions and work with joint distributions.
- Meet the mathematical demands of the core Ph.D. courses, especially microeconomics, macroeconomics, and econometrics.

## Suggested Textbooks

You are not required to purchase a textbook. The following references are recommended by the faculty who will teach your first-year courses:

- Simon and Blume (1994), *Mathematics for Economists* (recommended by Professors Subha and Ellis)
- McKenna, Livernois, and Hoy (2001), *Mathematics for Economics*, 2nd edition (Professors Ellis

and Codrina)

- Chiang and Wainwright (2004), *Fundamental Methods of Mathematical Economics*, 4th edition (Professor Francisco)
- Sydsaeter and Hammond (1995), *Mathematics for Economic Analysis* (Professor Lozada)
- Professor Lozada's prerequisite handout

## Daily Schedule

Each session from Day 2 onward runs 9:00–10:30 AM, a break from 10:30–10:45 AM, 10:45 AM – 12:00 PM, and 12:00–1:00 PM.

| Day | Date | Topics |
|---|---|---|
| 1 | July 28<br>1–4 PM | **Assessment and foundations.** Anonymous skills assessment based on Professor Lozada's handout (pp. 1, 3, 4) to identify knowledge gaps; set theory and notation; function notation $f : \mathbb{R}^n \to \mathbb{R}$; domain and range; summation notation and double sums; review of assessment results and preview of upcoming topics. |
| 2 | July 29 | **Linear algebra fundamentals.** Matrix definition and basic operations; multiplication and transposition; symmetric matrices; determinants and inverses; rank and linear independence; practice problems. |
| 3 | July 30 | **Linear algebra applications.** Systems of linear equations (Gaussian elimination, Cramer's rule); eigenvalues and eigenvectors (characteristic equations, diagonalization); economic applications: input–output models and Markov chains. |
| 4 | July 31 | **Calculus review I.** Limits and continuity; derivatives and rules; optimization problems and elasticity; the fundamental theorem of calculus and basic integration techniques. |
| 5 | August 1 | **Multivariable calculus.** Partial and higher-order partial derivatives; the gradient vector and the multivariable chain rule; marginal products and rates of substitution. |
| 6 | August 4 | **Optimization theory.** Unconstrained optimization (first- and second-order conditions, the Hessian); constrained optimization (Lagrange multipliers, Kuhn–Tucker conditions); utility maximization and cost minimization. |
| 7 | August 5 | **Concavity and convexity.** Convex sets and convex hulls; concave and convex functions, tests, and second-order conditions; applications to producer and consumer theory. |
| 8 | August 6 | **Probability and statistics.** Probability density and cumulative distribution functions; joint, marginal, and conditional distributions; expected value and variance; proof by contradiction. |
| 9 | August 7 | **Differential equations I.** Separable and linear first-order equations; integrating factors and substitution methods; applications to growth models and market dynamics. |

| Day | Date | Topics |
|---|---|---|
| 10 | August 8 | **Differential equations II.** Homogeneous second-order equations and characteristic equations; systems of differential equations (matrix and eigenvalue methods); phase diagrams, stability analysis, and Jacobian matrices. |
| 11 | August 11 | **Dynamic systems.** Difference equations and stability conditions; continuous-time optimization and steady states; applications to economic growth and business cycles. |
| 12 | August 12 | **Optimal control theory.** Setting up the Hamiltonian and necessary conditions; the current-value Hamiltonian and transversality conditions; optimal consumption and saving, and resource economics. |
| 13 | August 13 | **Advanced topics.** Inverse functions and the implicit function theorem; moment generating functions and limit theorems; advanced integration and applications to probability. |
| 14 | August 14 | **Review and practice.** Comprehensive review of tools for microeconomics and macroeconomics; mixed and exam-style problems; Q&A and exam preparation. |
| 15 | August 15 | **Final exam.** Comprehensive exam covering all topics, with a focus on applications to economics. |

**Emphasis by field.** Microeconomics: optimization and concavity/convexity (Days 6–7). Macroeconomics: differential equations and dynamic systems (Days 9–12). Econometrics: probability and statistics foundations (Day 8).

## Policies and Expectations

- Attendance is strongly encouraged because of the fast pace.
- Participation is encouraged: ask questions, help your peers, and engage actively.
- Homework is optional but will be reviewed in class.
- The final exam is diagnostic and formative. It will help you identify your strengths and weaknesses before the semester begins.

## Supplementary Materials and Tools

- Professor Lozada's old exam problems, useful for additional practice on prerequisite math topics.
- Selected slides from Professor Rada on dynamic models, the Hamiltonian, and optimal control theory (optional).
- Software for exploring matrices and functions: Mathematica, Python, or R.
- Reference management and notes: Zotero and Obsidian.
- Writing: learn LaTeX (for example, on Overleaf); a good starting guide is Oetiker's *The Not So Short Introduction to LaTeX 2$_\varepsilon$*.
- AI tools such as Claude or ChatGPT for brainstorming, summarizing, or language help.
- University resources: cloud storage, Microsoft Office 365, Grammarly, over 200 software programs,

printing and scanning, and equipment checkout.